THE UNIVERSITY OF KANSAS SPACE TECHNOLOGY LABORATORIES
2291 Irving Hill Dr. -- Campus West Lawrence, Kansas 66044
Telephone:

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ANALYSIS OF CONDITIONS FOR OPERATING THE S193 RAD/SCAT IN SOLAR POINTING MODE

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James Pintar
Arun Sobti

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ANALYSIS OF CONDITIONS FOR OPERATING THE S193 RAD/SCAT IN SOLAR POINTING MODE

James Pintar and Arun Sobti
The University of Kansas Center for Research, Inc.
Lawrence, Kansas 66044

ABSTRACT

The S193 Rad/Scat, although initially programmed for operating in the earth pointing mode, can be operated in the solar pointing mode as well. The usual coordinate systems for describing the S193 in orbit are defined. The instructions for the operation of the Radiometer and Scatterometer are presented in terms of standard Euler angles for these coordinate systems. A sample analysis for the Scatterometer is described. The relationships between the various Euler angles and physically meaningful orbit parameters is defined.

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ANALYSIS OF CONDITIONS FOR OPERATING THE S193 RAD/SCAT IN SOLAR POINTING MODE

1.0 INTRODUCTION

The nominal attitude for operating the S193 is the earth pointing mode. However, under certain conditions it is possible to operate the S193 from the solar pointing disposition. For the radiometer, the condition is simply that the antenna boresight be beneath the horizon. For the scatterometer, a range restriction similar to the above condition must be true. In addition, the doppler return must be within the filter bands for that pitch angle. In general, if the vehicle is in solar pointing mode the latter condition is not met. However, by yawing the vehicle about the solar pointing axis and executing a roll or pitch of the antenna on the gimbal mount it is possible for some positions in the orbit to satisfy both conditions; the CTC mode is most generally applicable.

2.0 DEFINITIONS OF COORDINATE SYSTEMS

Earth Centered Coordinates:* The reference earth centered coordinate system is the Earth Centered Inertial (ECI) in which the origin is the center of the earth, the z-axis points toward the mean north pole of 1950.0, the x-axis points towards the vernal equinox of 1950, and the y-axis completes the right-handed coordinate system. A second Earth Centered Coordinate System is the Earth-Centered True (ECT) with the x-axis through the Greenwich meridian at GMT=0 and the z-axis through the north pole at GMT=0. Note that the z-axis of these two coordinates systems are not coincident. In general, the conversion from ECI to ECT will involve a nutation, a precession, and a rotation of the ECI coordinates. A third earth centered coordinate system can be defined as ECR (Earth Centered Rotating) which rotates with the earth, with the z-axis towards the north pole and the x-axis towards the Greenwich Meridian. ECR can be obtained from ECT by a rotation of the angular displacement of the earth since GMT=0.0 about the ECT z-axis. The ECR coordinate axis, shown in Figure 1, will be $\hat{X}$, $\hat{Y}$, $\hat{Z}$.

* SKYLAB Parameter Formulation Document; change 4, May 29, 1973.

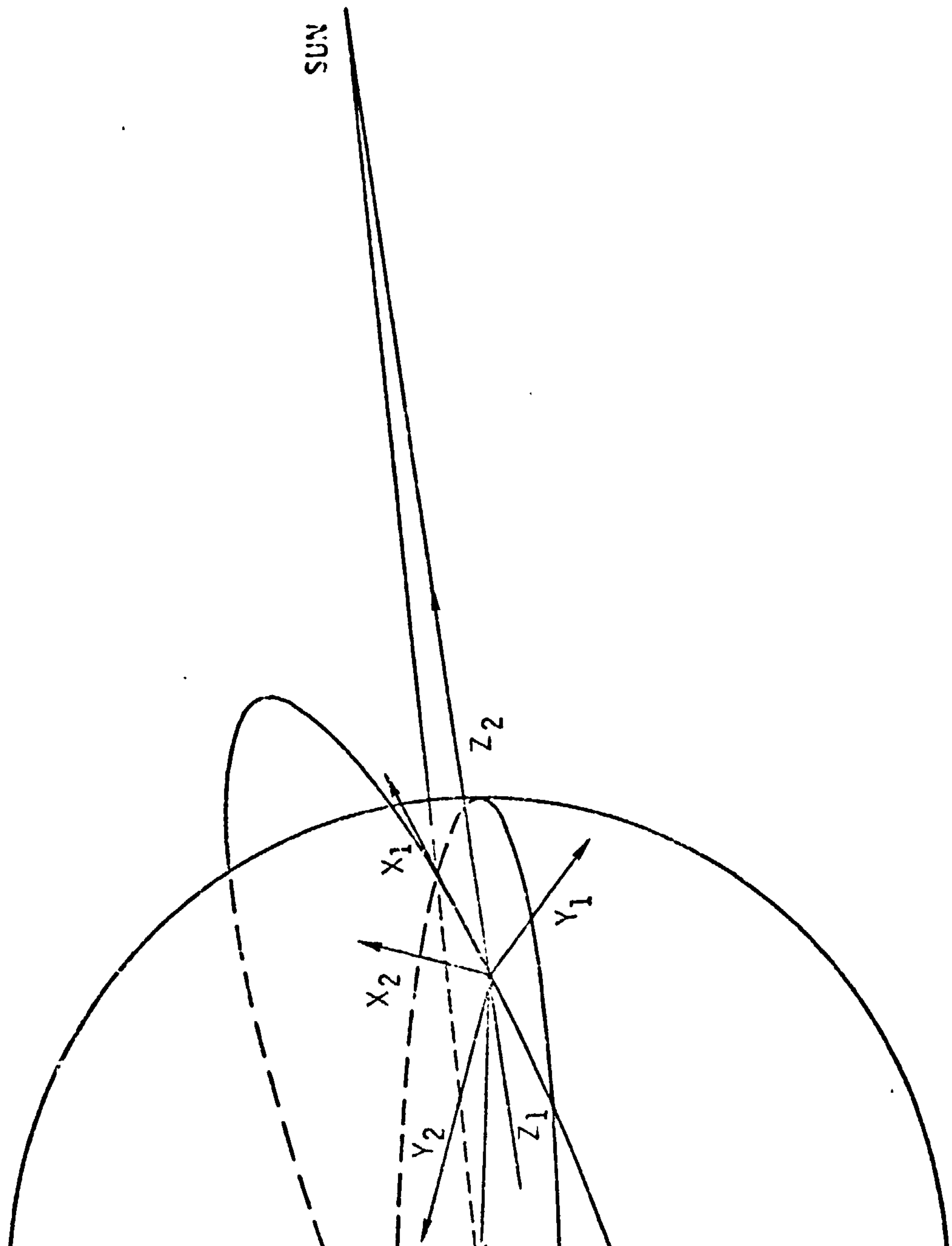

SUN
Z_2
X_1
Y_1
X_2
Y_2
Z_1
$\hat{Z}$
$\hat{X}$
GREENWICH MERIDIAN
$-\hat{Y}$
EQUATOR

Figure 1.

Z-Local Vertical (ZLV): The z-axis points from the center of spacecraft to the center of the earth, the x-axis points along the direction of the tangential component of the vehicle's velocity vector in the orbital plane, and the y-axis completes a right handed coordinate system. The ZLV unit vectors, shown in Figure 1, will be denoted as $\hat{X}_1$, $\hat{Y}_1$, $\hat{Z}_1$. For a circular orbit the x-axis and the velocity vector are colinear.

Solar Inertial Coordinates (SI): The z-axis points from center of spacecraft towards the sun, the x-axis lies in the vehicle's orbital plane and the y-axis completes a right handed coordinate system $\hat{X}_1'$, $\hat{Y}_1'$, $\hat{Z}_1'$.

Vehicle Coordinates (VEH) This coordinate system is defined in the vehicle itself with the x-axis along the long cylindrical axis of the vehicle $\hat{X}_2$, $\hat{Y}_2$, $\hat{Z}_2$. The conversion to VEH coordinates is achieved by the yaw, pitch, and roll attitudes errors from ZLV or SI coordinates.

Gimbal Mount Coordinates (GM): The gimbal mount coordinate system is centered at the gimbal mount and is defined by a misalignment error transformation in yaw, pitch, and roll from the VEH axes $\hat{X}_3$, $\hat{Y}_3$, $\hat{Z}_3$.

Antenna Line of Sight Axes (ANT): This coordinate system is centered at the antenna with the z-axis pointing in the direction of the antenna boresight: roll being a rotation about the GM x-axis, and pitch being a rotation about the GM y-axis: $\hat{X}_4$, $\hat{Y}_4$, $\hat{Z}_4$.

3.0 ANALYSIS

The S193 has four data taking modes of operation.* In the in-track non-contiguous mode (ITNC), the antenna scans in five discrete steps beginning with a pitch angle of 48° and ending at nadir. The scan period is 15.25 seconds. There is no roll offset.

In the in-track contiguous mode (ITC), the antenna scans continuously from 48° to nadir with 0° roll angle. The scan time including flyback is approximately 4.0 seconds.

For the cross-track non-contiguous mode (CTNC), the antenna scans in five discrete steps beginning with a roll angle of 48° and ending at nadir; the pitch

*SKYLAB program, EREP Investigators' Information Book, MSC-07874, April 1973.

offset is 0°. The scan cycle execution time is 15.25 seconds. The antenna slew can be either to the left, the right, or left then right of nadir.

In the cross-track contiguous mode (CTC), there are two submodes. (1) With a pitch offset of 0°, the antenna scans $\pm$ 11.375° about a roll offset of 0°, $\pm$ 15.6°, $\pm$ 29.4°. The scan is in roll plus a pitch variation from $+1^\circ$ to -1° during the scan to insure that the target cells lie in the cross-track plane. The scan time is 2.2 seconds. (2) With a roll offset of 0°, the antenna executes a roll excursion of $\pm$ 11.375° about a pitch offset of 0.0°, 15.6°, 29.4°, or 40.1°. The $+1^\circ$ to -1° pitch variation occurs for the reason sighted above. The scan time is 2.2 seconds.

The RAD and SCAT are operated jointly in the ITNC, ITC, and CTNC modes. In the CTC mode the data acquisition can be operated RAD only, SCAT only, or jointly.

The transformation from VEH to GM coordinate systems involves the mechanical misalignment error. These errors were established during the installation of the S193 onto the SKYLAB vehicle and have a maximum value of 0.15° for each of the three Euler angles. In the analysis to follow, these misalignment errors will be ignored. Also, the VEH to GM (A_3) transformation involves a translation from the center of the vehicle to the center of the Gimbal Mount. Again, since this distance is small compared to the range of the antenna, it will be ignored.

The other source of error usually associated with the normal operation of the S193 is the vehicle attitude errors, which indicate the misalignment of the vehicle centered coordinates from ZLV. For the SI operation a similar type of attitude error is present.* In general, this error can involve angles of significant magnitude. To compensate for this deviation another Euler angle transformation could be introduced separately, or incorporated with the attitude error transformation from VEH to ZLV to obtain an exact analysis. The critical point in these attitude errors would be the accuracy with which they can be described and also their predictability. All that is required is that they be known to within $\pm 1^\circ$ one day before the anticipated use of the S193 SCAT, so that the available doppler angle width can be correctly adjusted.

* Private communication with Aldo Bardano NASA-JSC-FCD.

3.1 Euler Angle Transformations

We can move the antenna boresight from one coordinate system to another by applying an Euler angle transformation. A general rotation is defined in the following manner.*

Begin with the coordinate axis $x_1\ y_1\ z$.

Rotate an angle ζ about the z-axis to get ξ, η, z.

Rotate an angle θ about the y-axis to get ξ, η, z',

Finally, rotate about the z-axis by an angle ψ to get x', y', z'.

For all three rotations, a positive angle is a rotation in the counter clockwise direction. Let A denote the transformation matrix.

A = BCD

$$D = \begin{pmatrix} \cos\zeta & \sin\zeta & 0 \\ -\sin\zeta & \cos\zeta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

$$B = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If R' is some vector defined in the primed coordinate system, we can re-express it in terms of the unprimed coordinates by applying the inverse transformation

$$\vec{R} = A^{-1}\vec{R}'$$
$$A^{-1} = A^T$$
$$\vec{R} = A^T\vec{R}'$$

Let A_1 represent the transformation from ECR to ZLV. A_1 is defined by the Euler angles $\zeta_1, \theta_1, \psi_1$, and is physically related to the latitude, longitude, and orbital inclination angle of the vehicle. Let A_2 represent the transformation ZLV to VEH. A_2 is defined by the Euler angles $\zeta_2, \theta_2, \psi_2$, and is related to the sun angle, latitude and longitude of the sub-solar point together with the SI to VEH attitude errors, or solely by the ZLV ro VEH attitude errors. Likewise A_3 and A_4 are defined by rotations into the GM and ANT coordinate systems. In general if we want to express a vector defined in one coordinate system in terms of a second coordinate system we simply premultiply by the proper transformation.

*Goldstein, Herbert; Classical Mechanics, pp. 107-109, Addison Wesley; Reading, Massachusetts, 1965.

$$A^t = \begin{pmatrix} \cos\psi\cos\zeta - \cos\theta\sin\zeta\sin\psi & -\sin\psi\cos\zeta - \cos\theta\sin\zeta\cos\psi & \sin\theta\sin\zeta \\ \cos\psi\sin\zeta + \cos\theta\cos\zeta\sin\psi & -\sin\psi\sin\zeta + \cos\theta\cos\zeta\cos\psi & -\sin\theta\cos\zeta \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{pmatrix}$$

For example the antenna boresight ($\hat{Z}_4$) in terms of ZLV coordinate would be

$$\hat{Z}_4' = A_2^t A_3^t A_4^t (\hat{Z}_4)$$

in general

$$\vec{R}_i' = A_j^t \ldots\ A_{i-1}^t\ A_i^t\ (\vec{R}_i)$$

The preceding results are summarized in the following table.

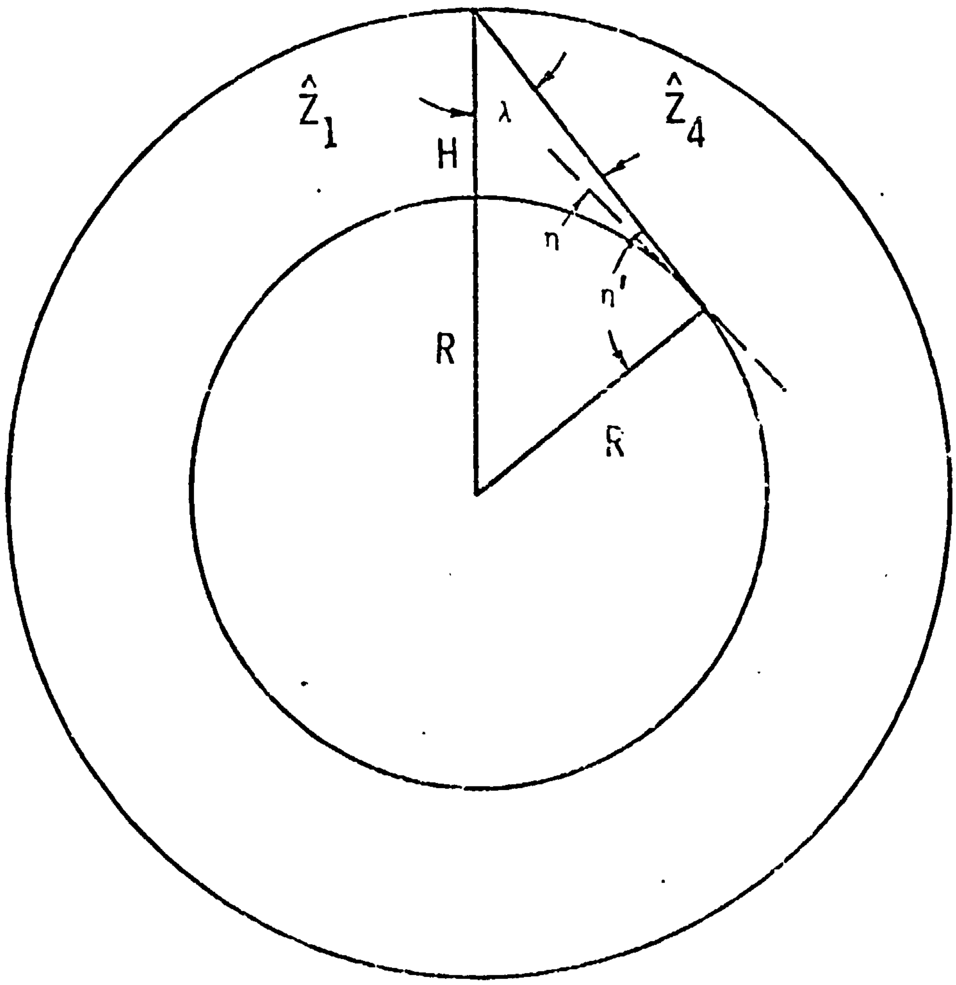
Coordinate System	Unit Vectors	Vector	Transformation to Preceeding Coordinates
ECI	$\hat{X}, \hat{Y}, \hat{Z}$	$\vec{R}$	
ZLV	$\hat{X}_1, \hat{Y}_1, \hat{Z}_1$	$\vec{R}_1$	$A_1(\zeta_1, \theta_1, \psi_1)$
VEH	$\hat{X}_2, \hat{Y}_2, \hat{Z}_2$	$\vec{R}_2$	$A_2(\zeta_2, \theta_2, \psi_2)$
GM	$\hat{X}_3, \hat{Y}_3, \hat{Z}_3$	$\vec{R}_3$	$A_3(\zeta_3, \theta_3, \psi_3)$
ANT	$\hat{X}_4, \hat{Y}_4, \hat{Z}_4$	$\vec{R}_4$	$A_4(\zeta_4, \theta_4, \psi_4)$

Similarly, these four coordinate transformation could be defined in terms of the usual yaw, pitch, and roll transformations. Yaw being a rotation about the z-axis; pitch, a rotation about the transformed y-axis; and roll, a rotation about the transformed x-axis.

$$A = RPY$$

or $\vec{R}' = Y^{-1} P^{-1} R^{-1} \vec{R}.$

In general, the type of matrix used to define a transformation is determined by the convenience of incorporating the available ephemeris data.



Earth radius: R ≈ 6371 Km.
Orbital Height: H ~ 435 Km

Figure 2.

3.2 Conditions for Radiometer Operation

The only criterion for operation of the S193 radiometer while the SKYLAB is in solar pointing disposition is that the antenna LOS vector must be below the horizon. The problems of doppler filters and range gates attendant to the Scatterometer are not present for operation of the passive device. To optimize the performance of the Radiometer, a minimum earth incidence angle is desired. This effects a reduction in the size of the resolution cell. This can be achieved by operating the Radiometer in a CTC mode in conjunction with a vehicle yaw about the solar axis. In the CTNC, ITC, or ITNC modes of operation, the antenna will sweep to a maximum angle of 48.0° on each scan. The optimum choice will depend upon the attitude of the vehicle relative to the sun and earth.

In the event of no or partial cloud cover, it would be desirable to have the S190 camera employed simultaneously with the S193 Radiometer to provide ground truth information. Also, the cameras would unambiguously identify exactly where the radiometer antenna was pointed since the center of the S190 picture would locate the GM z-axis. This would be particularly helpful if the exact attitude of the vehicle with respect to ZLV could not be determined.

Let η be the angle between the antenna boresight and the tangent at surface of the earth at the point of incidence, see Figure 2. Assuming that the earth is a sphere, to minimize the angle of incidence implies maximizing η. Cos (λ) is the direction cosine between $\hat{Z}_1$ and $\hat{Z}_4$.

$$\cos \lambda = \hat{z}_1 \cdot \hat{z}_4$$

$$\frac{\sin \lambda}{R} = \frac{\sin \eta'}{R+H} \qquad \eta' = \eta + 90^{\circ}$$

$$\frac{\sin \lambda}{R} = \frac{\sin(\eta + 90^{\circ})}{R+H} = \frac{\cos \eta}{R+H}$$

at the horizon $\eta = 0$

$$\sin \lambda_{MAX} = \frac{R}{R+H} = 0.9361 \qquad \text{For nominal } R \text{ and } H.$$

$$\lambda_{MAX} = 69.5^{\circ}$$

$$\lambda \leq 69.5^{\circ}$$

A_3 represents the misalignment between the vehicle coordinate axes and the GM axes. In general, these errors are small and can be ignored to a first approximation.

Consequently use A_3 to represent the SI to VEH attitude errors. Furthermore, assume that this error consists only of a yaw. A_2 will be the ZLV to SI transformation.

$$A_3^t = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

α is the yaw angle about the solar axis.

$$A_4^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_4+\tau) & -\sin(\theta_4+\tau) \\ 0 & \sin(\theta_4+\tau) & \cos(\theta_4+\tau) \end{pmatrix} \times \begin{pmatrix} \cos(\gamma+\varepsilon) & 0 & +\sin(\gamma+\varepsilon) \\ 0 & 1 & 0 \\ -\sin(\gamma+\varepsilon) & 0 & \cos(\gamma+\varepsilon) \end{pmatrix}$$

θ_4 is the antenna pitch angle offset

τ is the pitch excursion of $\pm$ 1° (applies only to CTC mode)

γ is the roll angle offset

ϵ is the roll excursion of $\pm$ 11.375° (applies only to CTC mode).

The general condition is shown in Appendix I, and is written for refere low:

$$\begin{aligned} \cos\lambda = {} & \sin\theta_2 \sin\Psi_2 \left[+\cos\alpha \sin(\gamma+\varepsilon) - \sin\alpha \sin(\theta_4+\tau) \cos(\gamma+\varepsilon) \right] \\ & + \sin\theta_2 \cos\Psi_2 \left[\pm\sin\alpha \sin(\gamma+\varepsilon) + \cos\alpha \sin(\theta_4+\tau) \cos(\gamma+\varepsilon) \right] \\ & + \cos\theta_2 \left[-\cos(\theta_4+\tau) \cos(\gamma+\varepsilon) \right] \end{aligned}$$

For ITNC

$\tau = \gamma = \varepsilon = 0°$

θ_4 = 0°, 15.6°, 29.4°, 40.1°, 48°

$$\cos\lambda = -\sin\theta_2 \sin\Psi_2 \sin\alpha \sin\theta_4 + \sin\theta_2 \cos\Psi_2 \cos\alpha \sin\theta_4 - \cos\theta_2 \cos\theta_4$$

For ITC the restraints are same except θ_4 is now continuous.

For CTNC

$$\tau = \theta_1 = \varepsilon = 0^\circ$$

$$\gamma = 0^\circ,\ 15.6^\circ,\ 29.4^\circ,\ 40.1^\circ,\ 48.0^\circ$$

$$\cos\lambda = +\sin\theta_2 \sin\psi_2 \cos\alpha \sin\gamma + \sin\theta_2 \cos\psi_2 \sin\alpha \sin\gamma - \cos\theta_2 \cos\theta_4 \cos\gamma$$

For CTC

$$\gamma = 0^\circ;\quad \tau = \pm 1^\circ;\quad \varepsilon = \pm 11.275^\circ;\quad \theta_4 = 0^\circ,\ 15.6^\circ,\ 29.4^\circ,\ 40.1^\circ$$

$$\cos\lambda = \sin\theta_2 \sin\psi_2 \left[+\cos\alpha \sin\varepsilon - \sin\alpha \sin(\theta_4+\tau)\cos\varepsilon\right] + \sin\theta_2 \cos\psi_2 \times \left[\pm\sin\alpha \sin\varepsilon + \cos\alpha \sin(\theta_4+\tau)\cos\varepsilon\right] - \cos\theta_2 \cos(\theta_4+\tau)\cos\varepsilon$$

$$\theta_4 = 0^\circ;\quad \gamma = 0^\circ,\ \pm 15.6^\circ,\ \pm 29.4^\circ$$

$$\cos\lambda = \sin\theta_2 \sin\psi_2 \left[+\cos\alpha \sin(\gamma+\varepsilon) - \sin\alpha \sin\tau \cos(\gamma+\varepsilon)\right] + \sin\theta_2 \cos\psi_2 \times \left[\pm\sin\alpha \sin(\gamma+\varepsilon) + \cos\alpha \sin\tau \cos(\gamma+\varepsilon)\right] - \cos\theta_2 \cos(\gamma+\varepsilon)\cos\tau$$

The Euler angles θ_2 and ψ_2 are functions of the vehicle orbital position and the position of the sun.

One simple criterion for selecting the optimum mode is to select that which minimizes the resolution cell, that is, minimizes the angle λ. Another possible criterion would be to select λ, such that the incidence angles for the radiometer lie within a specified bound, i.e. angle λ is equal to a threshold value.

3.3 Conditions for Scatterometer Operation

The two conditions affecting the operation of the scatterometer in SP mode are the doppler bandwidth and the range gates. The range gate sets an upper limit on the angle λ (as defined in Section 3.2) which will be less than the 69.5° limit established by the earth's radius and the orbital height.

As mentioned in the introduction, the second restriction on the operation of the scatterometer pertains to the doppler filter banks. In general, if the antenna boresight is oriented such that there is a component of signal propagation along the direction of motion, the frequency of the signal will be doppler shifted by an amount proportional to the cosine of the angle between the antenna LOS and the direction of motion. There will be a similar shift for the signal return. To provide for this, the SCAT employs filter bands each with a finite bandwidth centered on the doppler

shifted frequency. The selection of the filter bank used is determined by the pitch angle of the antenna LOS. The bandwidth of the filter bank defines a doppler angle width about the pitch angle. Any signal which is transmitted and received beyond these limits is attenuated excessively.

Consider the following example: Suppose that the antenna has been moved to a pitch position of 15.6°. Assume that the doppler angle width is $\pm 5^{\circ}$. If the vehicle is in the ZLV position then the doppler angle will be the complement of the pitch angle or 74.4°. For the SP operation the centroid of the doppler filter banks is still determined by the pitch angle of the antenna on the Gimbal Mount. However, this pitch angle is no longer equal to the complement of the doppler angle (the angle between antenna LOS and the direction of motion). In the SP mode the only degree of freedom available which can correct this discrepancy is a yaw about the solar axis. This procedure is more readily applied to the CTC mode of operation. It must be insured that the entire $\pm 11.375^{\circ}$ roll excursion lies within the doppler angle width.

The doppler bandwidths and doppler angle widths are determined by the 3 dB points of the filter banks (given in the table below).

Pitch Angle	Doppler Bandwidth*	Filter Bandwidth	Angle Width
0.0°	17.055	153	5.45°
15.6°	16.627	149.6	5.54°
29.4°	15.040	138.2	5.66°
40.1°	13.200	125.4	5.895°
48.0°	11.562	111.2	6.00°

Column 4 of this table includes a margin for the antenna beamwidth, which is taken to be 1.5°. For the sake of convenience, the doppler angle width can be assumed to be constant for all pitch angles at $\pm 5.5^{\circ}$.

Once again, ignoring the misalignment errors between the vehicle and the gimbal mount, the doppler condition requires that the angle between the antenna boresight and the direction of motion be equal to the complement of the antenna pitch angle. Here we have assumed a circular orbit.

* For a beamwidth of 1.54°.

$$\hat{z}_4' \cdot \hat{x}_1 = \sin\theta_4$$

$$\hat{x}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\hat{z}_4' = A_2^{+} A_3^{+} A_4^{+}(\hat{z}_4)$$

The general relationship is shown in Appendix I, and is written below for reference:

$$\begin{aligned}\sin\sigma = \hat{x}_1\cdot\hat{z}_4' = &\left[\cos\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\sin\psi_2\right]\left[+\cos\alpha\sin(\gamma+\epsilon) - \sin\alpha\sin(\theta_4+\tau)\right.\\ &\left.\times\cos(\gamma+\epsilon)\right] + \left[-\sin\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\cos\psi_2\right]\\ &\times\left[+\sin\alpha\sin(\gamma+\epsilon) + \cos\alpha\sin(\theta_4+\tau)\cos(\gamma+\epsilon)\right]\\ &+\left[\sin\theta_2\sin\zeta_2\right]\left[-\cos(\theta_4+\tau)\cos(\gamma+\epsilon)\right]\end{aligned}$$

For ITNC and ITC

$\tau = \gamma = \epsilon = 0^\circ$; $\theta_4 = 48^\circ \rightarrow 0^\circ$

$$\begin{aligned}\sin(\theta_4+\delta) \geq &\left[\cos\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\sin\psi_2\right]\left[-\sin\alpha\sin\theta_4\right] + \left[-\sin\psi_2\cos\zeta_2\right.\\ &\left.-\cos\theta_2\sin\zeta_2\cos\psi_2\right]\left[\cos\alpha\sin\theta_4\right] + \sin\theta_2\sin\zeta_2\left[-\cos\theta_4\right]\\ &\geq \sin(\theta_4-\delta)\end{aligned}$$

For fixed ψ_2, ζ_2, θ_2, α, this relationship must be true for θ_4 from 0° through 48°.

δ is the doppler angle width as defined in the table above.

For CTNC

$\tau = \theta_4 = \epsilon = 0^\circ$

$\gamma = \pm 48^\circ \rightarrow 0^\circ$ or $+48^\circ \rightarrow -48^\circ$

$$\begin{aligned}\sin\delta \geq \Big| &\left[\cos\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\sin\psi_2\right]\left[+\cos\alpha\sin\gamma\right]\\ &+\left[-\sin\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\cos\psi_2\right]\left[+\sin\alpha\sin\gamma\right]\\ &+\left[\sin\theta_2\sin\zeta_2\right]\left[-\cos\gamma\right]\Big|\end{aligned}$$

For fixed ψ_2, ζ_2, θ_2, and τ, this relationship must hold for all γ's.

For CTC

$\gamma = 0^\circ$

$$\sin(\theta_4+\delta) \geq [\cos\psi_2 \cos\zeta_2 - \cos\theta_2 \sin\zeta_2 \sin\psi_2][+\cos\alpha \sin\varepsilon - \sin\alpha \sin(\theta_4+\tau)\cos\varepsilon]$$
$$+[-\sin\psi_2 \cos\zeta_2 - \cos\theta_2 \sin\zeta_2 \cos\psi_2][+\sin\alpha \sin\varepsilon + \cos\alpha \sin(\theta_4+\tau)\cos\varepsilon]$$
$$+[\sin\theta_2 \sin\zeta_2][-\cos(\theta_4+\tau)\cos\varepsilon] \geq \sin(\theta_4-\delta)$$

Must hold for

$$\varepsilon = -11.375^\circ; \quad \tau = +1^\circ$$

and

$$\varepsilon = +11.375^\circ; \quad \tau = -1^\circ$$

For CTC

$\theta_4 = 0$

$$\sin\delta \geq \Big|[\cos\psi_2 \cos\zeta_2 - \cos\theta_2 \sin\zeta_2 \sin\psi_2][+\cos\alpha \sin(\gamma+\varepsilon) - \sin\alpha \sin\tau \cos(\gamma+\varepsilon)]$$
$$+[-\sin\psi_2 \cos\zeta_2 - \cos\theta_2 \sin\zeta_2 \cos\psi_2][+\sin\alpha \sin(\gamma+\varepsilon) + \cos\alpha \sin\tau \cos(\gamma+\varepsilon)]$$
$$+[\sin\theta_2 \sin\zeta_2][-\cos\tau \cos(\gamma+\varepsilon)]\Big|$$

Must hold for

$$\gamma \geq 0^\circ$$
$$\varepsilon = +11.375^\circ; \quad \tau = +1^\circ$$
$$\varepsilon = -11.375^\circ; \quad \tau = -1^\circ$$

and for

$$\gamma \leq 0^\circ$$
$$\varepsilon = -11.375^\circ; \quad \tau = +1^\circ$$
$$\varepsilon = +11.375^\circ; \quad \tau = -1^\circ$$

Inspection of these conditions indicates that the CTC zero pitch mode is most generally useful, while the use of ITC and ITNC is marginal.

The range condition on the scatterometer is determined by the transmit-receive timing sequence. The maximum angle λ (as defined in Figure 2) for full signal integration time is 54.1° for a pitch angle less than 48°, and 55.9° for a pitch angle of 48°. See Appendix II for details.

3.3.1 Simplified Analysis of Scat Operation in CTC 0° Pitch Mode

A detailed analysis has been carried out for the operation of the Scatterometer in the CTC 0° pitch mode. In this treatment the mechanical misalignment errors, as well as the $\pm$ 1° pitch excursion have been ignored. Furthermore, this analysis was performed using coordinate axis different from those defined in Section 2.0 of this paper. Other than confusing the point, this discrepancy makes no difference in the results. The results of the analysis are parameterized in terms of two angles which are independent of the coordinate system used to define them.

These two angles were μ, the sun angle and Ω (this angle will be used again in Section 3.4 with the same definition), the angle between the direction of motion and the x-axis of the vehicle. In standard coordinates the second angle is defined as

$$\cos\Omega = \hat{X}_1 \cdot \hat{X}_2'$$

Since the Scat has a zero pitch offset, the doppler angle will be 90°. This leads to the condition for determining the yaw necessary to achieve this condition.

$$-\tan\gamma = \frac{\cos\mu \sin\Omega}{\sin\Omega\sqrt{\cos^2\Omega - \cos^2\mu}\,\cos\alpha + \cos^2\Omega \sin\alpha}$$

The requirement that the roll excursion remain within the doppler width angle leads to

$$\sin\delta \geq \left| \frac{\sin\varepsilon \tan\Omega \cos\mu}{\sin\gamma} \right|$$

and the range condition is expressed as

$$\cos\lambda_{MAX} \leq \cos\lambda = \left[\sqrt{\cos^2\Omega - \cos^2\mu}\,\cos\alpha - \sin\Omega\sin\alpha\right]\sin\gamma + \cos\mu\cos\gamma$$

$$\lambda_{MAX} = 54.1039^\circ;\ \delta = 5.5^\circ$$

A computer program was written to investigate the usable region in Ω, μ space for Scat operation which would satisfy the above conditions.

The range of the two parameters were

$$0^\circ < \Omega \leq 90^\circ$$

$$0^\circ < \mu \leq 90^\circ$$

this range of μ implies that the local time as determined at the vehicle was restricted to the period from 6:00 to 18:00 hours. It is the range restriction which renders the

other 12 hours in the vehicle day useless. The range of Ω from 0 to -90° is a mirror image of the one investigated. Figure 3 and Figure 4, show the results of the study. The shaded area indicates the usable values of μ and Ω. The doppler restriction can be solved analytically and is shown as one of the bounds on the usable area. The range restriction was more complicated and the boundary shown was determined numerically. The program stepped through the angles in one degree increments and is therefore precise to one degree. However, it was designed to illustrate the capability of the Scat rather than to provide usable information for a real time operation.

The listing of the program appears in Section 3.3.2, as well as a sample of the output which includes the yaw required to achieve zero doppler shift and the earth incidence angle of the resulting antenna LOS.

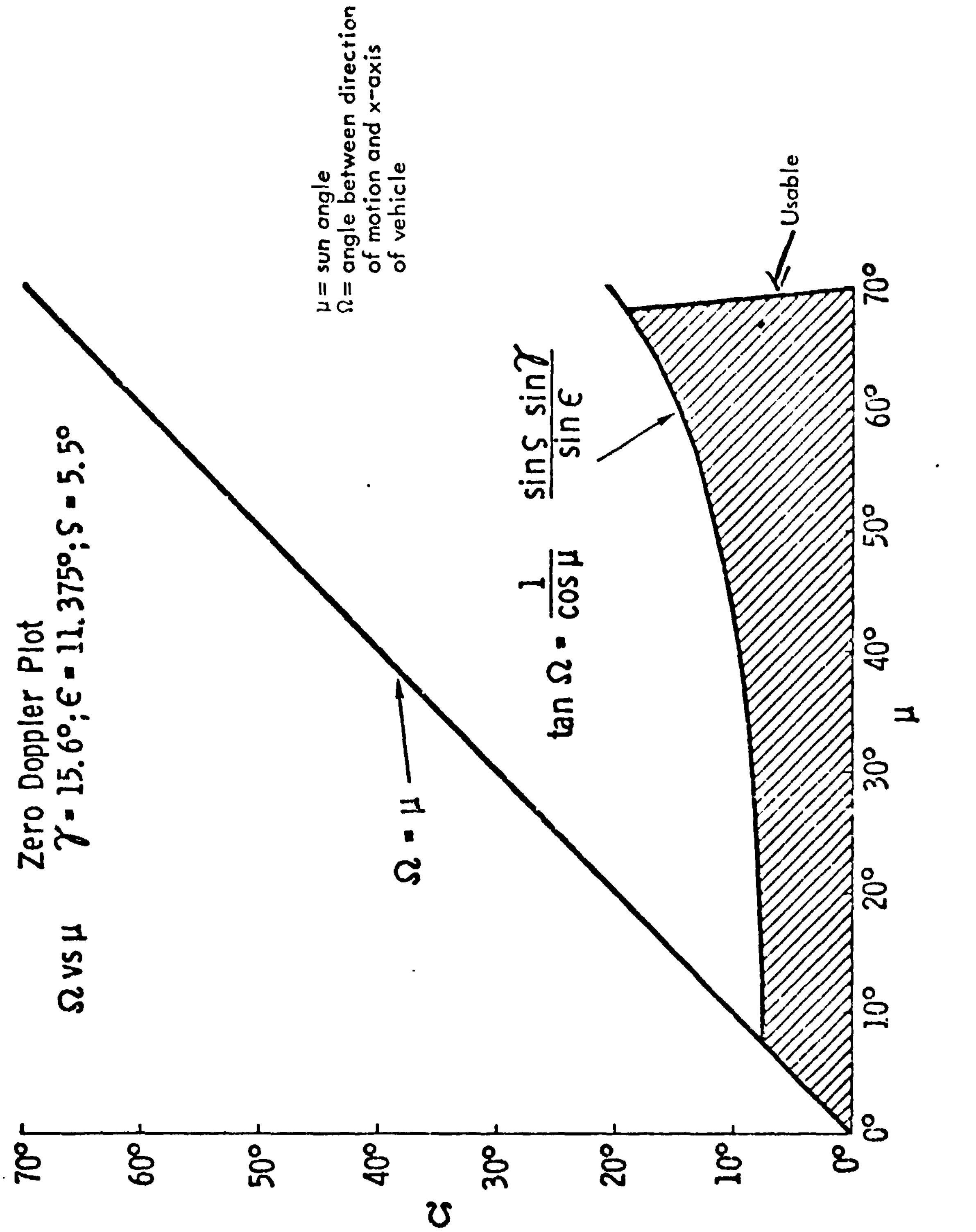


Figure 3. Values of Ω and μ For Which S193 Scat Can Be Used In CTC Pitch 0°, Roll 15.6 Mode.

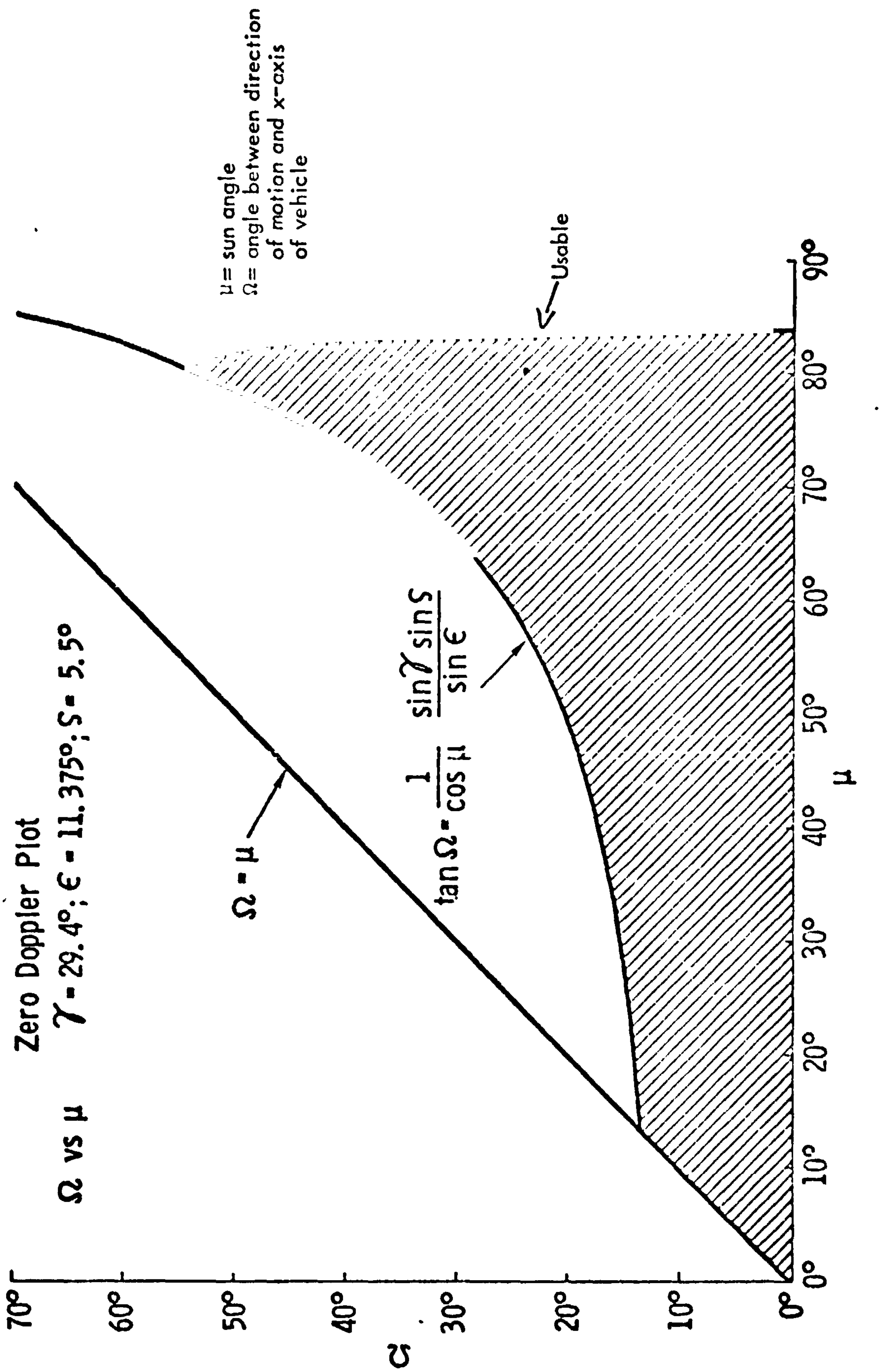


Figure 4. Values of Ω and μ For Which S193 Scat Can Be Used in CTC Pitch 0°, Roll 29.4° Mode.

```
CZDFSP     ZERO DOPPLER FROM SOLAR POINTING MODE                          00000040
           REAL MU,LAMMAX                                                 00000050
      C    THIS PROGRAM CALCULATES THE CONDITION FOR ZERO DOPPLER         00000060
      C    SHIFT WHEN THE VEHICLE IS IN SOLAR POINTING MODE,              00000070
      C    IN ZLV THE Z            AXIS IS ALONG THE LINE FROM THE CENTER 00000080
      C    OF THE EARTH TO THE VEHICLE,UP BEING THE POSITIVE DIRECTION,   00000090
      C    THE Y-AXIS IS THE DIRECTION OF MOTION .                        00000100
      C    IN S.P., THE Z-AXIS IS POSITIVE TOWARDS THE SUN.               00000110
      C    ***********************************************************    00000120
           DIMENSION G(2),DEL(2),ALPHA(2)                                 00000130
      C    MU IS THE SUN ANGLE,THAT IS, THE ANGLE BETWEEN ZLVZ-AXIS       00000140
      C          AND THE S.P.Z-AXIS.                                      00000150
      C    OMEGA IS THE ANGLE BETWEEN THE ZLV Y-AXIS AND S,P, Y-AXIS,     00000160
      C    GAMMA IS THE ROLL ANGLE OF THE ANTENNA ON ITS GIMBAL MOUNT,    00000170
      C    EPSIL IS THE ANTENNA EXCURSION,                                00000180
      C    SLOP IS THE MAXIMUM ANGLE FOR ZERO DOPPLER SHIFT,              00000190
      C          IT IS PROPORTIONAL TO THE DOPPLER BANDWIDTH,             00000200
      C    ALPHA IS THE YAW ANGLE.                                        00000210
      C     LAMBDA IS THE ANGLE BETWEEN BORESIGHT AND ZLV Z-AXIS AFTER    00000220
           DATA G/15.6,29.4/                                              00000230
      C          VEHICLE YAW AND ANTENNA ROLL,                            00000240
      C    ETA IS THE ANGLE OF INCIDENCE OF THE BEAM ON THE SURFACE OF THE EARTH,0250
      C    THE SITUATION WHERE EITHER MU OR OMEGA IS ZERO IS IGNORED      00000260
      C          IN THE PROGRAM,                                          00000270
           DATA RDTDG/57,2958/,EP/11,375/,SLOP/5.5/                       00000280
           TAN(X)=SIN(X)/COS(X)                                           00000290
           DGTRD = 1.0/RDTDG                                              00000300
           LAMMAX = 54.1039 * DGTRD                                       00000310  2
           EPSIL = DGTRD*EP                                               00000320  3
           SLOPDP = DGTRD*SLOP                                            00000330  4
           DELMAX =SIN(SLOPDP)                                            00000340  5
           WRITE(6,199)SLOP,DELMAX                                        00000350  6
       199 FORMAT(10X,'DOPPLER SLOP ANGLE = ',F10.5,10X                   00000360  9
          1 'DELMAX = ',F10,5)                                            00000370
           R = 6371.0                                                     00000380  9
           H = 435.0                                                      00000390  10
           WRITE(6,198)R,H                                                00000400  11
       198 FORMAT(1X,'ASSUME EARTH RADIUS OF ',F7.1,                      00000410  14
          1'KILOMETERS AND ORBITAL HEIGHT OF',F7.1,'KILOMETERS'//)        00000420
           WRITE(6,200)                                                   00000430  14
       200 FORMAT(1H0,10X,'    MU        OMEGA      ALPHA   ',            00000440  16
          1'   GAMMA      DELTA      DELTA    SINETA      ETA    '//)     00000450
           DO 10 I = 1,90                                                 00000460  16
           M=I                                                            00000470  17
           IF(M.EQ.0)GOTO10                                               00000480  18
           MU = DGTRD*FLOAT(M)                                            00000490  21
           JLO=1                                                          00000500  22
           JHI = I                                                        00000510  23
           DO 9 J = JLO,JHI                                               00000520  24
           IOM =J                                                         00000530  25
           IF(IOM.EQ.0)GO TO 9                                            00000540  26
           OMEGA = DGTRD * FLOAT(IOM)                                     00000550  29
```

3.3.2 Program Listout and Sample Output.

```
            IF(IABS(IOM).NE.IABS(M)) GO TO 499                        00000560  30
            RTB=0.00                                                  00000570  33
            GO TO 500                                                 00000580  34
     499    CONTINUE                                                  00000590  35
            RTB = COS(OMEGA)**2-COS(MU)**2                            00000600  36
            IF(RTB .GT.0.0)GO TO 500                                  00000610  37
            IE = 1000                                                 00000620  40
            PRINT 1101,IE,M,IOM,I,J,MU,OMEGA,RTB                      00000630  41
    1101    FORMAT(1X,5I5,5F10.5)                                     00000640  44
            GO TO 9                                                   00000650  44
     500    B = SQRT(RTB)                                             00000660  45
            DO 8 IG = 1,2                                             00000670  46
            GAMMA = G(IG)*DGTRD                                       00000680  47
      C     CALCULATE THE VEHICLE YAW.                                00000690
            RTC=TAN(GAMMA)**2*(COS(OMEGA)**2/COS(MU)**2               00000700  48
           1 /SIN(OMEGA)**2-1.0) - 1.0                                00000710
            IF(RTC.GT.0.00) GO TO 501                                 00000720  49
            GO TO 8                                                   00000730  52
     501    C = COS(OMEGA)/TAN(OMEGA)*SQRT(RTC)                       00000740  53
            A=COS(MU)*SIN(OMEGA)**2/TAN(GAMMA)/(COS(OMEGA)**2         00000750  54
           1-SIN(OMEGA)**2*COS(MU)**2)                                00000760
            COSALP = A*(-1.0*B+C)                                     00000770  55
            COSALM=A*(-1.0*B-C)                                       00000780  56
            IF((ABS(COSALP).GT.1.0).OR.(ABS(COSALM).GT.1.0))GOTO1003  00000790  57
            SINALM = SQRT(1.0-COSALM**2)                              00000800  60
            SINALP=SQRT(1.0-COSALP**2)                                00000810  61
            INDEX=0                                                   00000820  62
    9999    INDEX=INDEX+1                                             00000830  63
            IF(INDEX.EQ.2)GOTO2                                       00000840  64
       1    COSAL=COSALM                                              00000850  67
            SINAL=SINALM                                              00000860  68
            GO TO 3                                                   00000870  69
       2    COSAL = COSALP                                            00000880  70
            SINAL=SINALP                                              00000890  71
       3    IND =0                                                    00000900  72
         4  IND=IND+1                                                 00000910  73
            CHECK=COS(MU)/(B*COSAL+COS(OMEGA)/TAN(OMEGA)*             00000920  74
           1 SINAL)                                                   00000930
            IF(ABS(CHECK+TAN(GAMMA)).LT.0.00001)GO TO 5               00000940  75
            IF(IND.GE.2)GO TO 1004                                    00000950  78
            SINAL=-SINAL                                              00000960  81
            GOTO 4                                                    00000970  82
    1004    WRITE(6,1005) I,IOM,IG,INDEX,CHECK,A,B,C,COSAL,SINAL,COSALM,COSALP 00000980  83
    1005    FORMAT(20X,20(1H*)/1X,4I4,8F10.4/20X,20(1H*))             00000990  86
       5    ALPHA(INDEX)=ATAN2(SINAL,COSAL)                           00001000  85
            IF(INDEX.EQ.1)GOTO 9999                                   00001010  87
      C     SEE IF EXCURSION EXCEEDS DOPPLER SLOP                     00001020
            DO7IP=1,2                                                 00001030  90
            PHASE =-1.0                                               00001040  91
            IF(MOD(IP,2).EQ.0)PHASE=1.0                               00001050  92
            SINDEL=ABS(SIN(EPSIL)*TAN(OMEGA)*COS(MU)*(-1.0*           00001060  99
           1 COS(GAMMA)/TAN(GAMMA)+PHASE*SIN(GAMMA)))                 00001070
```

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```
          IF(SINDEL.GT.SIN(SLOPDP))GOTO8                                     00001080  96
          DEL(IP)=SINDEL                                                     00001090  99
    7     CONTINUE                                                           00001100 100
          DGAM=GAMMA*RDTDG                                                   00001110 102
          DOMEGA=OMEGA*RDTDG                                                 00001120 103
          DMU=MU*RDTDG                                                       00001130 104
          DO71IND=1,2                                                        00001140 105
          AL =ALPHA(IND)*RDTDG                                               00001150 106
          COSLAM=(B*COS(ALPHA(IND))-SIN(OMEGA)*SIN(ALPHA(IND)))*SIN(GAMMA)   00001160 107
         1 + COS(MU)*COS(GAMMA)                                              00001170
          IF(ABS(COSLAM).LE.1.0000)GO TO 74                                  00001180 108
          WRITE(6,1006)M,IOM,IG,IND,COSLAM,A,B,C                             00001190 111
 1006     FORMAT(20X,20(1H*)/1X,'COSLAM GREATER THAN ONE, SET EQUAL TO ONE'  00001200 114
         1 /1X,4I5,4F10.4/20X,20(1H*))                                       00001210
          COSLAM = 1.0                                                       00001220 114
   74     CONTINUE                                                           00001230 115
          IF(COS(LAMMAX).GT.ABS(COSLAM))GO TO 71                             00001240 116
          SINLAM = SQRT(1.0 - COSLAM**2)                                     00001250 119
          SINETA = (R+H)/R*SINLAM                                            00001260 120
          IF(ABS(SINETA).GT.1.0)GO TO 71                                     00001270 121
          ETA = ATAN2(SINETA,SQRT(1.0-SINETA**2))*RDTDG                      00001280 124
          WRITE(6,201)DMU,DOMEGA,AL,DGAM,DEL,SINETA,ETA                      00001290 125
  201     FORMAT(10X,8F10.4)                                                 00001300 126
   71     CONTINUE                                                           00001310 128
    8     CONTINUE                                                           00001320 130
    9     CONTINUE                                                           00001330 132
   10     CONTINUE                                                           00001340 134
          STOP                                                               00001350 136
 1003     IE=1003                                                            00001360 137
          PRINT 1100,IE,M,IOM,IG                                             00001370 138
          STOP                                                               00001380 141
 1100     FORMAT(1X,5I5)                                                     00001390 142
          END                                                                00001400 142
```

23756 WORDS OF MEMORY USED BY THIS COMPILATION

SNUMB = T1145, ACTIVITY # = 02, REPORT CODE = 06, RECORD COUNT = 04419

COPPLER SLOP ANGLE = 5.50000 DELMAX = 0.09985
ASSUME EARTH RADIUS OF 6371.0KILOMETERS AND ORBITAL HEIGHT OF 435.0KILOMETERS

MU	OMEGA	ALPHA	GAMMA	DELTA	DELTA	SINETA	ETA
1.0000	1.0000	-176.4158	15.6000	0.0128	0.0109	0.2857	16.6516
1.0000	1.0000	-3.5343	15.6000	0.0128	0.0109	0.2857	16.6516
1.0000	1.0000	-178.2249	29.4000	0.0070	0.0036	0.5242	31.6124
1.0000	1.0000	-1.7752	29.4000	0.0070	0.0036	0.5242	31.6124
2.0000	1.0000	-175.4476	15.6000	0.0128	0.0109	0.3177	18.5239
2.0000	1.0000	-3.6129	15.6000	0.0128	0.0109	0.2555	14.8319
2.0000	1.0000	-178.2559	29.4000	0.0070	0.0036	0.5521	33.5072
2.0000	1.0000	-1.8046	29.4000	0.0070	0.0036	0.4958	29.7219
2.0000	2.0000	-172.8131	15.6000	0.0255	0.0219	0.2850	16.5604
2.0000	2.0000	-7.1849	15.6000	0.0255	0.0219	0.2850	16.5604
2.0000	2.0000	-176.4469	29.4000	0.0140	0.0073	0.5234	31.5619
2.0000	2.0000	-3.5531	29.4000	0.0140	0.0073	0.5234	31.5619
3.0000	1.0000	-176.4595	15.6000	0.0128	0.0109	0.3372	19.7041
3.0000	1.0000	-3.6273	15.6000	0.0128	0.0109	0.2356	13.6258
3.0000	1.0000	-178.2764	29.4000	0.0070	0.0036	0.5695	34.7135
3.0000	1.0000	-1.8224	29.4000	0.0070	0.0036	0.4776	28.5281
3.0000	2.0000	-172.8787	15.6000	0.0255	0.0219	0.3250	18.9654
3.0000	2.0000	-7.2575	15.6000	0.0255	0.0219	0.2446	14.1587
3.0000	2.0000	-176.5277	29.4000	0.0140	0.0073	0.5594	34.0130
3.0000	2.0000	-3.6285	29.4000	0.0140	0.0073	0.4867	29.1207
3.0000	3.0000	-169.1813	15.6000	0.0384	0.0328	0.2822	16.3901
3.0000	3.0000	-10.8168	15.6000	0.0384	0.0328	0.2822	16.3901
3.0000	3.0000	-174.6633	29.4000	0.0210	0.0109	0.5221	31.4765
3.0000	3.0000	-5.3367	29.4000	0.0210	0.0109	0.5221	31.4765
4.0000	1.0000	-176.4715	15.6000	0.0128	0.0109	0.3556	20.8296
4.0000	1.0000	-3.6437	15.6000	0.0128	0.0109	0.2165	12.5061
4.0000	1.0000	-178.2765	29.4000	0.0070	0.0036	0.5859	35.8658
4.0000	1.0000	-1.6387	29.4000	0.0070	0.0036	0.4601	27.3928
4.0000	2.0000	-172.9493	15.6000	0.0255	0.0219	0.3468	20.2899
4.0000	2.0000	-7.2725	15.6000	0.0255	0.0219	0.2223	12.8414
4.0000	2.0000	-176.5743	29.4000	0.0140	0.0073	0.5788	35.3644
4.0000	2.0000	-3.6676	29.4000	0.0140	0.0073	0.4661	27.7843
4.0000	3.0000	-169.3316	15.6000	0.0383	0.0328	0.3295	19.2404
4.0000	3.0000	-10.9458	15.6000	0.0383	0.0328	0.2342	13.5466
4.0000	3.0000	-174.8077	29.4000	0.0210	0.0109	0.5647	34.3810
4.0000	3.0000	-5.4697	29.4000	0.0210	0.0109	0.4785	28.5872
4.0000	4.0000	-165.4959	15.6000	0.0512	0.0438	0.2781	16.1484
4.0000	4.0000	-14.5041	15.6000	0.0512	0.0438	0.2781	16.1484
4.0000	4.0000	-172.8713	29.4000	0.0280	0.0145	0.5204	31.3569
4.0000	4.0000	-7.1288	29.4000	0.0280	0.0145	0.5204	31.3569
5.0000	1.0000	-176.5143	15.6000	0.0128	0.0109	0.3736	21.9353
5.0000	1.0000	-3.6546	15.6000	0.0128	0.0109	0.1978	11.4059
5.0000	1.0000	-178.3168	29.4000	0.0070	0.0036	0.6018	36.9954
5.0000	1.0000	-1.8541	29.4000	0.0070	0.0036	0.4427	26.2773
5.0000	2.0000	-172.9982	15.6000	0.0255	0.0218	0.3665	21.4969
5.0000	2.0000	-7.3217	15.6000	0.0255	0.0218	0.2018	11.6425
5.0000	2.0000	-176.6182	29.4000	0.0140	0.0072	0.5962	36.5958
5.0000	2.0000	-3.7016	29.4000	0.0140	0.0072	0.4473	26.5693
5.0000	3.0000	-169.4176	15.6000	0.0383	0.0328	0.3535	20.7024
5.0000	3.0000	-11.0016	15.6000	0.0383	0.0328	0.2095	12.0935
5.0000	3.0000	-174.8860	29.4000	0.0210	0.0109	0.5860	35.8745
5.0000	3.0000	-5.5332	29.4000	0.0210	0.0109	0.4558	27.1154

5.0000	4.0000	-165.7262	15.6000	0.0511	0.0437	0.3319	19.3856
5.0000	4.0000	-14.6735	15.6000	0.0511	0.0437	0.2236	12.9212
5.0000	4.0000	-173.0910	29.4000	0.0280	0.0145	0.5686	34.6560
5.0000	4.0000	-7.3288	29.4000	0.0280	0.0145	0.4707	28.0781
5.0000	5.0000	-161.7389	15.6000	0.0639	0.0547	0.2728	15.8317
5.0000	5.0000	-18.2612	15.6000	0.0639	0.0547	0.2728	15.8317
5.0000	5.0000	-171.0678	29.4000	0.0350	0.0181	0.5181	31.2023
5.0000	5.0000	-8.9323	29.4000	0.0350	0.0181	0.5181	31.2023
6.0000	1.0000	-176.5380	15.6000	0.0127	0.0109	0.3913	23.0347
6.0000	1.0000	-3.6683	15.6000	0.0127	0.0109	0.1791	10.3178
6.0000	1.0000	-178.3374	29.4000	0.0070	0.0036	0.6173	38.1223
6.0000	1.0000	-1.6688	29.4000	0.0070	0.0036	0.4254	25.1771
6.0000	2.0000	-173.0476	15.6000	0.0255	0.0218	0.3852	22.6575
6.0000	2.0000	-7.3470	15.6000	0.0255	0.0218	0.1821	10.4917
6.0000	2.0000	-176.6615	29.4000	0.0140	0.0072	0.6127	37.7878
6.0000	2.0000	-3.7330	29.4000	0.0140	0.0072	0.4290	25.4042
6.0000	3.0000	-169.4935	15.6000	0.0382	0.0327	0.3745	21.9956
6.0000	3.0000	-11.0457	15.6000	0.0382	0.0327	0.1876	10.8111
6.0000	3.0000	-174.9575	29.4000	0.0209	0.0108	0.6046	37.1980
6.0000	3.0000	-5.5867	29.4000	0.0209	0.0108	0.4355	25.3143
6.0000	4.0000	-165.8540	15.6000	0.0510	0.0436	0.3580	20.9783
6.0000	4.0000	-14.7714	15.6000	0.0510	0.0436	0.1966	11.3408
6.0000	4.0000	-173.2259	29.4000	0.0279	0.0145	0.5918	36.2835
6.0000	4.0000	-7.4175	29.4000	0.0279	0.0145	0.4458	26.4755
6.0000	5.0000	-162.0612	15.6000	0.0638	0.0546	0.3325	19.4181
6.0000	5.0000	-18.5176	15.6000	0.0638	0.0546	0.2123	12.2595
6.0000	5.0000	-171.3734	29.4000	0.0350	0.0181	0.5715	34.3571
6.0000	5.0000	-9.2074	29.4000	0.0350	0.0181	0.4629	27.5733
6.0000	6.0000	-157.6666	15.6000	0.0767	0.0656	0.2561	15.4352
6.0000	6.0000	-22.1134	15.6000	0.0767	0.0656	0.2561	15.4352
6.0000	6.0000	-159.2497	29.4000	0.0420	0.0218	0.5152	31.0120
6.0000	6.0000	-10.7504	29.4000	0.0420	0.0218	0.5152	31.0120
7.0000	1.0000	-176.5526	15.6000	0.0127	0.0109	0.4088	24.1269
7.0000	1.0000	-3.6787	15.6000	0.0127	0.0109	0.1605	9.2345
7.0000	1.0000	-178.3565	29.4000	0.0070	0.0036	0.6326	39.2459
7.0000	1.0000	-1.8829	29.4000	0.0070	0.0036	0.4080	24.0817
7.0000	2.0000	-173.0984	15.6000	0.0254	0.0217	0.4035	23.7948
7.0000	2.0000	-7.3692	15.6000	0.0254	0.0217	0.1627	9.1660
7.0000	2.0000	-176.7651	29.4000	0.0139	0.0072	0.6257	38.9549
7.0000	2.0000	-3.7625	29.4000	0.0139	0.0072	0.4110	24.2655
7.0000	3.0000	-169.5791	15.6000	0.0382	0.0326	0.3942	23.2173
7.0000	3.0000	-11.0829	15.6000	0.0382	0.0326	0.1658	9.6023
7.0000	3.0000	-175.0270	29.4000	0.0209	0.0108	0.6218	38.4535
7.0000	3.0000	-5.6350	29.4000	0.0209	0.0108	0.4161	24.5909
7.0000	4.0000	-165.9722	15.6000	0.0509	0.0435	0.3804	22.3575
7.0000	4.0000	-14.8326	15.6000	0.0509	0.0435	0.1732	9.9763
7.0000	4.0000	-173.3038	29.4000	0.0279	0.0144	0.6115	37.6953
7.0000	4.0000	-7.4939	29.4000	0.0279	0.0144	0.4241	25.0934
7.0000	5.0000	-162.2372	15.6000	0.0637	0.0545	0.3606	21.1349
7.0000	5.0000	-18.6201	15.6000	0.0637	0.0545	0.1833	10.5600
7.0000	5.0000	-171.5226	29.4000	0.0349	0.0181	0.5964	36.6113
7.0000	5.0000	-9.3277	29.4000	0.0349	0.0181	0.4350	25.8496
7.0000	6.0000	-158.3128	15.6000	0.0765	0.0654	0.3312	19.3441
7.0000	6.0000	-22.4461	15.6000	0.0765	0.0654	0.2032	11.5465
7.0000	6.0000	-169.6510	29.4000	0.0419	0.0217	0.5735	34.9948
7.0000	6.0000	-11.1079	29.4000	0.0419	0.0217	0.4549	27.0610
7.0000	7.0000	-153.9112	15.6000	0.0894	0.0765	0.2580	14.9521
7.0000	7.0000	-26.0889	15.6000	0.0894	0.0765	0.2580	14.9521
7.0000	7.0000	-167.4139	29.4000	0.0490	0.0254	0.5118	30.7851
7.0000	7.0000	-12.5862	29.4000	0.0490	0.0254	0.5118	30.7851

3.4 Relationship Between Euler Angles and Skylab Ephemerides

In order to use the preceding relationships effectively it is necessary to express the Euler angles for transformations A_1 and A_2 in terms of available orbit information.

Often, and in particular, for post flight analysis, the SKYLAB ephemerides data will be available or at least obtainable. In this case all the angles necessary to completely describe the position and orientation of the vehicle would be available. A detailed analysis could then be conducted. However, for preflight analysis, accurate SKYLAB Data is generally not available. What is presented below is a first order approximation of the ECR to ZLV to SI transformations based on the assumption of a circular orbit with nominal inclination and reasonab[illegible]owledge of the vehicle's anticipated orbit in terms of latitude, longitude, and [illegible]

The transformation A_1 from ECR to ZLV coordinate system has been defined in terms of the Euler angles ζ_1, θ_1, ψ_1. These can be further defined with respect to the vehicle's latitude, longitude and orbital inclination angle, Figure 1.

In terms of the Euler angles the ZLV unit vectors in ECR coordinates are

$$\hat{X}_1' = \begin{pmatrix} \cos\psi_1 \cos\zeta_1 - \cos\theta_1 \sin\zeta_1 \sin\psi_1 \\ \cos\psi_1 \sin\zeta_1 + \cos\theta_1 \cos\zeta_1 \sin\psi_1 \\ \sin\theta_1 \sin\psi_1 \end{pmatrix}$$

$$\hat{Y}_1' = \begin{pmatrix} -\sin\psi_1 \cos\zeta_1 - \cos\theta_1 \sin\zeta_1 \cos\psi_1 \\ -\sin\psi_1 \sin\zeta_1 + \cos\theta_1 \cos\zeta_1 \cos\psi_1 \\ \sin\theta_1 \cos\psi_1 \end{pmatrix}$$

$$\hat{Z}_1' = \begin{pmatrix} \sin\theta_1 \sin\zeta_1 \\ -\sin\theta_1 \cos\zeta_1 \\ \cos\theta_1 \end{pmatrix}$$

In terms of spherical polar coordinates they can be expressed as

$$\hat{X}_1' = \begin{pmatrix} \sin a \cos b \\ \sin a \sin b \\ \cos a \end{pmatrix}$$

$$\hat{Y}_1' = \begin{pmatrix} \sin c \cos d \\ \sin c \sin d \\ \cos c \end{pmatrix}$$

$$\hat{Z}_1' = \begin{pmatrix} \sin e \cos f \\ \sin e \sin f \\ \cos e \end{pmatrix}$$

By equating the entries of these two vectors we find.

$$\cos\theta_1 = \cos e$$
$$\sin\theta_1 = \sin e$$

(Note: this choice of signs was arbitrary).

$$\cos\zeta_1 = -\sin f$$
$$\sin\zeta_1 = \cos f$$

$$\cos\psi_1 = \frac{\cos c}{\sin e}$$

(Note: sin(e) can never be zero for the SKYLAB orbit).

$$\sin\psi_1 = \pm\sqrt{1-\cos\psi_1^2}$$

(Sign determined by ascending or descending pass).

$e = 90^\circ$ + latitude (North latitudes are positive; south latitude are negative)

$f = 180^\circ$ + longitude (East longitudes are positive; west longitudes are negative)

$c = 180^\circ -$ orbital inclination angle

Expressing the Euler angles of A_2 in terms of the orbit ephemerides produces additional difficulties. To facilitate this discussion, assume that the ray from the sun to the center of the earth is parallel to the ray from the sun to the vehicle. The validity of this approximation is demonstrated in Appendix III. This implies that the two rays (the second of which is Z_2) have the same orientation in ECR coordinates.

Consider the following equivalent representation of the transformation A_2^+:

rotate 180° about $\hat{X}_1$ to get $\hat{X}_1, \hat{Y}_1, \hat{Z}_1$.

rotate an angle Ω about $\hat{Y}_1$ to get $\hat{X}_2, \tilde{\hat{Y}}_1, \tilde{\hat{Z}}_1'$ (this leaves X_2 in the orbital plane)

finally rotate an angle ζ about $\hat{X}_2$ to get $\hat{X}_2, \hat{Y}_2, \hat{Z}_2$

In terms of these rotations

$$A_2^+ = \begin{pmatrix} \cos\Omega & +\sin\Omega\sin\rho & +\sin\Omega\cos\rho \\ 0 & -\cos\rho & \sin\rho \\ +\sin\Omega & -\cos\Omega\sin\rho & -\cos\Omega\cos\rho \end{pmatrix}$$

By equating the entries of the two forms for A_2^+ we find

$$\cos\theta_2 = -\cos\Omega\cos\rho \qquad \sin\theta_2 = +\sqrt{1-\cos^2\Omega\cos^2\rho}$$

$$\cos\zeta_2 = \frac{-\sin\rho}{\sin\theta_2} \qquad \sin\zeta_2 = \frac{+\sin\Omega\cos\rho}{\sin\theta_2}$$

$$\cos\psi_2 = \frac{-\cos\Omega\sin\rho}{\sin\theta_2} \qquad \sin\psi_2 = \frac{+\sin\Omega}{\sin\theta_2}$$

The angles Ω and ζ can now be expressed in terms of the sun angle (μ), the sun's latitude and longitude, and the other orbit information mentioned above. The sun angle is actually a redundant, though useful parameter. How this angle is related to the other ephemerides is demonstrated in Appendix IV. The sun's latitude and longitude actually refer to the location of the point of incidence at the surface of the earth with the ray joining the centers of the earth and sun.

Denoting $\hat{Z}_2$ in terms of ECI spherical polar coordinates as $\hat{Z}_2''$

$$A_1 \hat{z}_2'' = \hat{z}_2' = \begin{pmatrix} -\sin\Omega\cos\rho \\ \sin\rho \\ -\cos\Omega\cos\rho \end{pmatrix}$$

where $\hat{Z}_2'$ is the same unit vector expressed in ZLV coordinates. From the sun's latitudes and longitudes we define

$i = 90^{\circ} -$ latitude	North latitude is positive South latitude is negative
$j =$ longitude	East longitude is positive West longitude is negative

Then

$$\hat{z}_2'' = \begin{pmatrix} \sin i \cos j \\ \sin i \sin j \\ \cos i \end{pmatrix}$$

Using the definition that

$$\hat{z}_1 \cdot \hat{z}_2' = -\cos\mu$$

or $$-\cos\mu = -\cos\Omega \cos\rho$$

This equation plus the equality of the x and y components of the vector above yield after some algebra

$$\sin\rho = \frac{\sin i}{\sin e}\left[\cos a \sin(f-j) - \cos e \cos c \cos(f-j)\right] + \cos c \cos i$$

$$\cos a = \sin e \sin \psi_1$$

$$\cos\rho = \pm\sqrt{1-\sin^2\rho}$$

$$\cos\Omega = \cos\mu / \cos\rho$$

$$\sin\Omega = \pm\sqrt{1-\cos^2\Omega}$$

where i, j, e, f, c, ψ_1, are defined earlier in this section. The signs of the functions can be determined by inspection of the sun unit vector in ZLV coordinates.

3.5 Minimal Input Simulation Program

To achieve the capability of providing pre-flight analysis for solar pointing operation, a simulation program was developed. Essentially, the program consists of a stripped down version of the SKYLAB simulation program available at the University of Kansas. It incorporates the transformations from ANT to ECR coordinates and tests the doppler and range conditions for SCAT mode. The program inputs consist of the predicted latitude, longitude, and GMT of the vehicle, the sun's declination and equation of time (approximate numbers are obtainable from any physical geography text), the antenna pitch and roll angles desired, and the user's best guess as to what the vehicle attitude errors will be. To provide approximate analyses from this minimum of input data several assumptions were made. These included a circular orbit, spherical earth, and nominal earth radius, orbital radius, and orbital inclination angle (although the latter parameters can be convienently varied). The important features of the program are contained into subroutines: SETA2 and SETA2. The listings appear in the following sections.

```
CSETA1      CONSTRUCT A1 TRANSFORMATION
      SUBROUTINE SETA1
      REAL LAT,LONG,NEXTLA
C      ZLV TO ERC TRANSFORMATION (EARTH CENTERED ROTATING)
      COMMON/ANGLE1/LAT,LONG,OINC,    NEXTLA,SINSIG
      COMMON/VEC/RE,H,RVEH,COSLAM,VEH,VECTOR
C      ALL ANGLES IN RADIANS
      DIMENSION A(3,3),VECTOR(3)
      DIMENSION VEH(3)
      DATA PI/3.1415927/
      E = PI / 2.0 + LAT
      F = PI + LONG                                                2
      C = PI - OINC                                                3
      COSTHE = COS(E)                                              4
      SINTHE = SIN(E)                                              5
      COSPHI = -SIN(F)                                             6
      SINPHI = COS(F)                                              7
      COSPSI = COS(C) / SINTHE                                     8
      SINPSI = SQRT(1.0 - COSPSI * COSPSI )                        9
      IF(NEXTLA .LT.LAT) SINPSI = -1.0 * ABS(SINPSI)              10
      A(1,1) = COSPSI*COSPHI - COSTHE*SINPHI*SINPSI               13
      A(1,2) = -SINPSI*COSPHI - COSTHE*SINPHI*COSPSI              14
      A(1,3) = SINTHE*SINPHI                                      15
      A(2,1) = COSPSI*SINPHI + COSTHE*COSPHI*SINPSI               16
      A(2,2) = -SINPSI*SINPHI + COSTHE*COSPHI*COSPSI              17
      A(2,3) = -SINTHE*COSPHI                                     18
      A(3,1) = SINTHE*SINPSI                                      19
      A(3,2) = SINTHE*COSPSI                                      20
      A(3,3) = COSTHE                                             21
      VTHE = PI/2.0 - LAT                                         22
      RF1 = SIN(VTHE)* COS(LONG)                                  23
      RF2 = SIN(VTHE) * SIN(LONG)                                 24
      RF3 = COS(VTHE)                                             25
      RETURN                                                      26
      ENTRY A1                                                    27
      VEH(1) = RF1                                                27
      VEH(2) = RF2                                                28
      VEH(3) = RF3                                                29
      COSLAM = VECTOR(3)                                          30
      SINSIG = VECTOR(1)                                          31
      CALL MH (A,VECTOR,1)                                        32
      RETURN                                                      33
      END                                                         34

23826 WORDS OF MEMORY USED BY THIS COMPILATION
```

3.5.1 Listout of SETA1.

```
CSETA2          A2 FOR SI MODE
      SUBROUTINE SETA2
      COMMON/ANGLE2/PHI,THETA,PSI,GMT                   ,SUNDEC,EQTIME
     1, RMU,ISWATT
      COMMON/ANGLE1/VLAT,VLONG,OINC,VNXTLA,SINSIG
      COMMON/VEC/RE,H,RVEH,COSLAM,VEH(3), VECTOR(3)
      DIMENSION A(3,3),AP(3,3),B(3,3),S(3)
      EQUIVALENCE (COSTHE,COSE),(SINTHE,SINE)
      DATA PI/3.1415927/,D/0.017453292/
C      SET UP THE VEH TO SI ATTITUDE ERROR TRANSFORMATION
      IF(ISWATT .EQ.0) GO TO 99
      CALL TRANS(PHI,THETA,PSI,A)                                        4
   99 AP(1,1) = 1.0                                                      5
      AP(1,2) = 0.0                                                      6
      AP(1,3) = 0.0                                                      7
      AP(2,1) = 0.0                                                      8
      AP(2,2) = -1.0                                                     9
      AP(2,3) = 0.0                                                     10
      AP(3,1) = 0.0                                                     11
      AP(3,2) = 0.0                                                     12
      AP(3,3) = -1.0                                                    13
      IF(ISWATT .EQ.0) GO TO 98                                         14
      CALL MH(A,A P,3)                                                  17
   98 E = PI/2.0 + VLAT                                                 18
      F = PI + VLONG                                                    19
      C = PI - OINC                                                     20
      COSE = COS(E)                                                     21
      SINE = SIN(E)                                                     22
      COSF = COS(F)                                                     23
      SINF = SIN(F)                                                     24
      COSC = COS(C)                                                     25
      SINC = SIN(C)                                                     26
      COSPS I = COS(C) / SINTHE                                         27
      SINPSI = SQRT(1.0 - COSPSI * COSPSI)                              28
      IF(VNXTLA.LT.VLAT) SINPSI = -1.0 * ABS(SINPSI)                    29
      COSA = SINE * SINPSI                                              32
      SINA = SQRT(1.0 - COSA * COSA)                                    33
      IF(VNXTLA .LT.VLAT) SINA = -1.0 * ABS(SINA)                       34
      RI = PI/2.0 - SUNDEC                                              37
      RJ = -1.0 * (GMT - 12.0 + EQTIME)*15.0 * D                        38
      COSPI = COS(RI)                                                   39
      SINRI = SIN(RI)                                                   40
      COSRJ = COS(RJ)                                                   41
      SINRJ = SIN(RJ)                                                   42
      COSRMU = -SINRI*SINE*(COSF*COSRJ+SINF*SINRJ)-COSE*COSRI           43
      SINRMU = SQRT(1.0 - COSRMU * COSRMU)                              44
      RMU = ATAN2(SINRMU,COSRMU)                                        45
      ARG = F - RJ                                                      46
      SINRHO = ABS(COSC*COSRI+SINRI/SINE*(COSA*SIN(ARG)-COSE*COSC*COS(ARG 47
     1G)))
      COSRHO = SQRT(1.0 - SINRHO * SINRHO)                              48
      COSOM = ABS(COSRMU/COSRHO)                                        49
```

3.5.2 Listout of SETA2.

```
          SINOM = SQRT(1.0 - COSOM * COSOM)                          50
          COSPHI = -SINF                                             51
          SINPHI =COSF                                               52
          B(1,1) = COSPSI*COSPHI - COSTHE*SINPHI*SINPSI              53
          B(2,1) = -SINPSI * COSPHI - COSTHE*SINPHI*COSPSI           54
          B(3,1) = SINTHE * SINPHI                                   55
          B(1,2) = COSPSI * SINPHI + COSTHE * COSPHI * SINPSI        56
          B(2,2) = -SINPSI * SINPHI + COSTHE * COSPHI * COSPSI       57
          B(3,2) = -SINTHE * COSPHI                                  58
          B(1,3) = SINTHE * SINPSI                                   59
          B(2,3) = SINTHE * COSPSI                                   60
          B(3,3) = COSTHE                                            61
          S(1) = SINRI * COSRJ                                       62
          S(2) = SINRI * SINRJ                                       63
          S(3) = COSRI                                               64
          PRINT 200,S                                                65
          CALL MM(B,S,1)                                             68
          PRINT 200,S                                                69
      200 FORMAT(1X,3E15.6)                                          72
          IF(S(3) .GT.0.0) GO TO 10                                  72
C          SX LT 0.0
          IF(S(2) .GT.0.0) GO TO 3                                   75
C          SY LT 0.0
          IF(S(1) .GT.0.0) GO TO 2                                   78
C           SX LT 0.0
          SINRHO = -SINRHO                                           81
          SINOM = -SINOM                                             82
          GO TO 20                                                   83
C          SX GT 0.0
        2 SINRHO = -SINRHO                                           84
          GO TO 20                                                   85
        3 IF(S(1) .GT.0.0) GO TO 4                                   86
C          SX LT 0.0
          SINOM = -SINOM                                             89
          GO TO 20                                                   90
C          SX GT 0.0
        4 GO TO 20                                                   91
C          SZ GT 0.0
       10 IF(S(2) .GT.0.0) GO TO 13                                  92
C          SY LT 0.0
          IF(S(1) .GT.0.0) GO TO 12                                  95
C          SX LT 0.0
          SINOM = -SINOM                                             98
          COSOM = -COSOM                                             99
          SINRHO = -SINRHO                                          100
          GO TO 20                                                  101
C          SX    GT 0.0
       12 COSOM = -COSOM                                            102
          SINRHO = -SINRHO                                          103
          GO TO 20                                                  104
C          SY GT 0.0
       13 IF(S(1) .GT.0.0) GO TO 14                                 105
```

```
105   C       SX LT 0.0
106          COSOM = -COSOM                              108
107          SINOM = -SINOM                              109
108          GO TO 20                                    110
109   C       SX GT 0.0
110       14 COSOM = -COSOM                              111
111          GO TO 20                                    112
112       20 A(1,1) = COSOM                              113
113          A(1,2) =  SINOM * SINRHO                    114
114          A(1,3) =  SINOM * COSRHO                    115
115          A(2,1) = 0.0                                116
116          A(2,2) = -COSRHO                            117
117          A(2,3) = SINRHO                             118
118          A(3,1) =  SINOM                             119
119          A(3,2) = -COSOM * SINRHO                    120
120          A(3,3) = -COSOM * COSRHO                    121
121          PRINT 200,A(1,3),A(2,3),A(3,3)              122
122          CALL MM(A,AP,3)                             125
123          RETURN                                      126
124          ENTRY A2                                    127
125          CALL MM(AP,VECTOR,1)                        127
126          RETURN                                      128
127          END                                         129

   23949 WORDS OF MEMORY USED BY THIS COMPILATION
```

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4.0 CONCLUSION

This analysis demonstrates the wide range of flexibility available for operating the S193 RAD/SCAT. Although originally designed to be specifically used in the ZLV mode, the instrument can also be used in the solar pointing mode. The RAD can successfully take data for greater than half of each orbit even in SP mode; the SCAT is functional over a somewhat smaller range.

Since the CTC mode has a RAD only option, continual RAD can be acquired from before vehicle sunrise to after vehicle sunset with a minimum of operator attention. Occasionally, the pitch or roll offset would have to be changed to maintain the antenna LOS below the horizon. Due to the complication of the doppler condition, the SCAT can only be operated over specific targets for a short duration of time during a non-ZLV pass. However, it seems reasonable that some scan mode can be at least partially successful in covering any target during a daylight pass.

This paper also presents the relationships between the Euler angles characterizing the ECR to ZLV and ZLV to SI transformations in terms of the orbit ephermides most readily available for preflight analysis. This facilitates determining the location of the target cells andl likelihood of satisfying the doppler condition on the SCAT.

What cannot be determined accurately preflight are the vehicle altitude errors, which in some cases can be quite sizable. However, an iterative approach spanning a range of yaw, pitch, and roll angles can be used to evaluate the probability for successful SCAT operation.

APPENDIX I

The direction cosines associated with the range and the doppler angle are defined as follows:

Let $\hat{Z}'_4$ represent the antenna boresight in ZLV coordinates

Let σ be the doppler angle

$$A_2^+ = \begin{pmatrix} \cos\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\sin\psi_2 & -\sin\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\cos\psi_2 & \sin\theta_2\sin\zeta_2 \\ \cos\psi_2\sin\zeta_2 + \cos\theta_2\cos\zeta_2\sin\psi_2 & -\sin\psi_2\sin\zeta_2 + \cos\theta_2\cos\zeta_2\cos\psi_2 & -\sin\theta_2\cos\zeta_2 \\ \sin\theta_2\sin\psi_2 & \sin\theta_2\cos\psi_2 & \cos\theta_2 \end{pmatrix}$$

$$A_3^+ = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ \sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$A_4^+ = \begin{pmatrix} \cos(\gamma+\varepsilon) & 0 & +\sin(\gamma+\varepsilon) \\ +\sin(\theta_4+\tau)\sin(\gamma+\varepsilon) & \cos(\theta_4+\tau) & -\sin(\theta_4+\tau)\cos(\gamma+\varepsilon) \\ -\cos(\theta_4+\tau)\sin(\gamma+\varepsilon) & \sin(\theta_4+\tau) & \cos(\theta_4+\tau)\cos(\gamma+\varepsilon) \end{pmatrix}$$

$$\begin{aligned} \cos\lambda = \hat{Z}_1 \cdot \hat{Z}'_4 = {} & \sin\theta_2\sin\psi_2\left[+\cos\alpha\sin(\gamma+\varepsilon) - \sin\alpha\sin(\theta_4+\tau)\cos(\gamma+\varepsilon)\right] \\ & + \sin\theta_2\cos\psi_2\left[+\sin\alpha\sin(\gamma+\varepsilon) + \cos\alpha\sin(\theta_4+\tau)\cos(\gamma+\varepsilon)\right] \\ & + \cos\theta_2\left[-\cos(\theta_4+\tau)\cos(\gamma+\varepsilon)\right] \end{aligned}$$

$$\begin{aligned} \sin\sigma = \hat{X}_1 \cdot \hat{Z}'_4 = {} & \left[\cos\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\sin\psi_2\right]\left[+\cos\alpha\sin(\gamma+\varepsilon) - \sin\alpha\sin(\theta_4+\tau)\right. \\ & \times \left.\cos(\gamma+\varepsilon)\right] + \left[-\sin\psi_2\cos\zeta_2 - \cos\theta_2\sin\zeta_2\cos\psi_2\right] \\ & \times\left[+\sin\alpha\sin(\gamma+\varepsilon) + \cos\alpha\sin(\theta_4+\tau)\cos(\gamma+\varepsilon)\right] \\ & + \left[\sin\theta_2\sin\zeta_2\right]\left[-\cos(\theta_4+\tau)\cos(\gamma+\varepsilon)\right] \end{aligned}$$

APPENDIX II

Maximum range as determined by scat transmit-receive timing. For full signal integration the first pulse transmitted must be the present over the entire integration slot.

TIME: T = (Scat Integrate start) - (Scat Transmit Start)

RANGE: R = 0.50 x T x C

Where C speed of light = 3×10^8 meters/sec.

Lambda: $\lambda = \arccos \left[\frac{(r+h)^2 + R^2 - r^2}{2(r+h)\,r} \right]$

h = orbital height ≈ 6371 km

r = earth radius ≈ 435 km

$\theta_4 \neq 48^{\circ}$*

T = 5.339 - 0.019 msec

R = 798 km

$\lambda = 54.1039^{\circ}$

$\theta_4 = 48^{\circ}$*

T = 5.649 - 0.019 msec

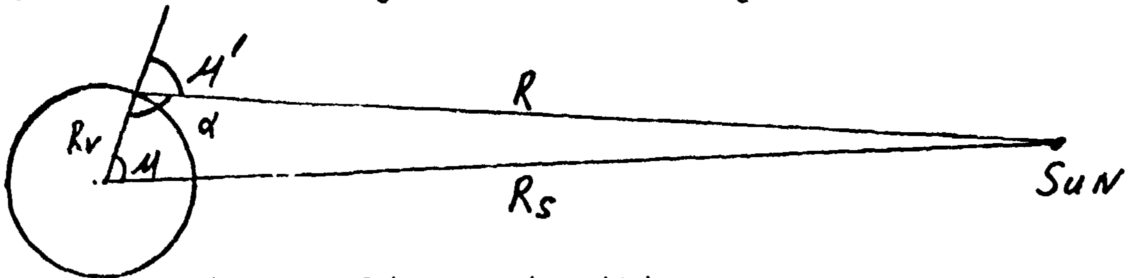
R = 844 km

$\lambda = 55.8894^{\circ}$

* G. E. Calibration Data Report Flight Hardware, Vol. 1A, RevD, 22 March 1973.

APPENDIX III

The definition of the sun angle μ is illustrated in the figure below



R is the distance from the center of the sun to the vehicle

R_v is the distance from the center of the earth to the vehicle

R_s is the distance from the center of the sun to the center of the earth

Using the law of sines and then the law at cosines we find

$$\frac{R}{\sin\mu} = \frac{R_s}{\sin\alpha} \; ; \qquad \alpha = 180^\circ - \mu'$$

$$R = \frac{R_s \sin\mu}{\sin\mu'}$$

$$R_s^2 = R_v^2 + R^2 + 2 R_e R \cos\mu'$$

$$\cos\mu' = \frac{R_s^2 - R_e^2 - R^2}{2 R_e R}$$

$$= \frac{\cos\mu - R_v/R_s}{\sqrt{1 + (R_v/R_s)^2 - 2 R_v/R_s \cos\mu}}$$

Now the cos μ' can be approximated by a Taylor series expansion

$$\cos\mu' = \cos\mu + (R_v/R_s)(1 + \cos\mu) + (R_v/R_s)^2 (3\cos^2\mu + 1) + \dots$$

The ratio (R_v/R_s) is approximately of the order of 4×10^{-5}.

APPENDIX IV

The sun angle can easily be determined from the vehicle latitude and longitude, the sun latitude and the Greenwich Mean Time.

$\theta_1 = 90^\circ$ - latitude of vehicle	North latitude is positive South latitude is negative
ζ_1 = longitude of vehicle	East longitude is positive West longitude is negative
$\theta_2 = 90^\circ$ - latitude of sun	Same convention as above

ζ_2 = longitude of sun = -(GMT-12.00 hr. and Delta) x 15 deg/hr.*

Using these definition in ECI coordinates

$$-\hat{Z}_1 = \begin{pmatrix} \sin\theta_1 \cos\zeta_1 \\ \sin\theta_1 \sin\zeta_1 \\ \cos\theta_1 \end{pmatrix}$$

$$\hat{Z}_2 = \begin{pmatrix} \sin\theta_2 \cos\zeta_2 \\ \sin\theta_2 \sin\zeta_2 \\ \cos\theta_2 \end{pmatrix}$$

$$\mu = \arccos\left[\sin\theta_1 \sin\theta_2 \cos(\zeta_1 - \zeta_2) + \cos\theta_1 \cos\theta_2\right].$$

* Delta = difference between the apparent and mean solar times, often referred to as the equation of time. Delta varies from +17 to -17 minutes during the year.